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Group schemes and rigidity of algebras in positive characteristic

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Abstract

Rigidity criteria for a finite dimensional associative or Lie algebra of positive characteristic are given. A geometrically rigid algebra may have deformations with nontrivial infinitesimals which may be interpreted as obstructions to integrating infinitesimal automorphisms. A group scheme theoretic nature of those obstructions is revealed. For each affine group scheme G of finite type over the ground field an invariantly defined G-module $\mathrm{Obs}(G)$ is introduced and formal properties of the functor $G \mapsto \mathrm{Obs}(G)$ are studied.

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Introduction

Let L be a finite dimensional Lie algebra over a field k. Gerstenhaber and Schack [8, 9] introduced the terminology which distinguishes several concepts of rigidity. It has long been known that L is both analytically and geometrically rigid provided that the second cohomology group $H^2(L, L)$ vanishes [6, 13]. For an algebraically closed k this result can be improved as follows. Denote by Sq the canonical quadratic mapping $H^2(L, L) \to H^3(L, L)$ (see [16]). Let $\mathrm{Ker}\,\mathrm{Sq} = \{\zeta \in H^2(L, L) \mid \mathrm{Sq}\,\zeta = 0\}$. Then L is both analytically and geometrically rigid provided that $\mathrm{Ker}\,\mathrm{Sq} = 0$. For geometric rigidity, one will find this result in a paper of Rauch [15, II, Theorem 3]. Further investigations showed that analytic rigidity implies geometric rigidity and that in the case of characteristic zero the converse is true [8]. Specific phenomena are encountered in characteristic $p > 0$. In this case $H^2(L, L)$ contains a subspace Obs which consists of obstructions to integrating the infinitesimal automorphisms. It was introduced by Gerstenhaber [7] in a more general context of composition complexes. We shall see that $\mathrm{Sq}\,\zeta = 0$ for $\zeta \in \mathrm{Obs}$. I do not know whether Sq factors through $H^2(L, L)/\mathrm{Obs}$. Now we shall summarize the main results of the paper.

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